HYDRODYNAMIC CHARACTERISTICS OF PONTONS PERTINENT TO A STUDY OF THEIR DESIGN AND OPERATION

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SYNOPSIS

Seven model pontons were tested to determine their hydrodynamic properties in vertical movement when partially submerged. The equivalent added weight due to the fluid was obtained by measuring changes in the natural fundamental frequency of a supporting beam for which the relationship between fundamental frequency and attached center-weight was known. The results are in agreement with previous tests and analyses where comparisons can be made.

The equivalent added weight was shown to have an important effect on the vibrational properties of pontons. Other studies** have shown that the equivalent added weight of the pontons has a significant effect on the response of floating bridges to transient loads.

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This work was sponsored by the Army, Navy, and Air Force through the Joint Services Advisory Committee for Research Groups in Applied Mathematics and Statistics by Contract No. Nonr-760(03). Dr.

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**Romualdi, reference 11 in appended bibliography

F. T. Mavis, Professor and Head, Department of Civil Engineering, was administrative supervisor. He and Dr. E. D'Appolonia, Associate Professor and technical supervisor of the contract, continually directed, reviewed, and discussed the progress of the study. Mr. J. P. Murtha directed most of the testing and computations. Helpful information on the design and use of pontons was provided by the Engineer Research and Development Laboratories, Fort Belvoir, Virginia.

INTRODUCTION

This report describes a study conducted under contract Nonr-760(03), Office on Naval Research, Department of the Navy. The specific assignment under this contract was to conduct a basic analytical study of the design and performance of pontons. The study was initiated June 1, 1953 and completed August 31, 1954.

The total problem of design, analysis, fabrication, transport, assembly, testing, and operation of floating bridges was thoroughly reviewed. The most serious gap in present knowledge seemed to be the lack of information on the dynamic characteristics of the integral system of ponton and superstructure. Pontons and superstructures have usually been designed for hydrostatic forces under static loads. Individual pontons have usually been tested statically or in steady-state motion as towed rafts. Ponton bridges have been field-tested under moving loads, but the bulk of test data has often defied basic analysis of all pertinent factors. In operation, obviously, the forces, stresses, and behavior of floating bridges under the action of a moving load are essentially transient or dynamic.

The gap between static-design and dynamic-operation is usually covered with an "impact factor." Thus, static stresses have been increased perhaps fifteen percent to allow for additional stress due to dynamic action. Use of a blanket "impact factor" to cover ignorance of the real dynamic stresses is particularly undesirable when saving in weight and material is of importance.

To analyze the design and performance of pontons the forces acting on the ponton must be known. The first step in the present study, therefore, was to determine forces acting on a ponton for all typical conditions of vertical displacement, velocity, and acceleration.

Floating bridges are essentially a roadway or superstructure resting on floating supports or pontons whose reactions are primarily a function of changes in vertical displacement. If the superstructure is very flexible, the load is supported by only one or two pontons nearest the load. A more rigid superstructure, however, transfers part of the load to other more distant pontons. In common military design about forty percent of the load may be supported by one or two nearest pontons, and sixty percent by pontons farther away. The design of a floating bridge requires the selection of a balanced system of roadway flexibility and ponton capacity. As more of the load is transferred to distant pontons, the ponton capacity can be reduced, but roadway strength and rigidity must be increased. Because of the interaction of the ponton-superstructure system a study of the ponton alone would be of little value. The displacements, velocities, and accelerations in vertical movement are all affected by the system interaction.

Thus, the entire ponton-superstructure system must be studied to analyze the design and operation of pontons for both static and dynamic loads.

The behavior or operation of the floating bridges is further complicated because superstructure spans may be neither simply supported nor "fully continuous" in the sense that angle changes are not continuous linear functions of bending moments. Usually the connections are a combination of the two. Because of pin clearance for easy assembly, joints may rotate freely until the clearance is taken-up. The joint then locks and the structure is, in effect, continuous.

Knowledge of the behavior of the superstructure-substructure system under the action of stationary and moving loads is essential for the design and analysis of components of a floating bridge. Until the forces acting on the structure are known, no accurate analysis or design can be made, and the forces can only be obtained from a study of the whole system.

Scope of the Present Study

This report describes a study of the hydrodynamic properties in vertical movement of seven model pontons. The major part of the study deals with the acceleration response or virtual mass of the ponton since such information was almost totally lacking at the beginning of the study. The displacement response or buoyancy forces were measured and computed under static loads. The velocity response or drag forces were evaluated by use of experimental coefficients obtained from studies of damped vibration. The velocity response is less important than displacement or acceleration response since high velocities are not attainable in the limited displacement (3 to 4 ft) of the ponton.

If the response of the ponton is known for all displacements, velocities, and accelerations, the behavior of a floating bridge under the action of moving vehicle loads can be computed provided the structural properties of the bridge are known. To measure the accuracy with which behavior of a floating bridge can be computed once displacement-velocity-acceleration response is known, the model pontons were subjected to known impulsive loadings. The displacement of the ponton was then measured and compared with the computed displacement. Verification of the analysis with tests of specific models is sufficient to commend the analytical methods for use in design. However, corresponding field measurements on a prototype ponton would be desirable.

Once the analytical methods had been verified by model tests, other informative studies were made. Computation of vibrational properties and resonant frequencies of the ponton are presented. Evaluation of vehicle speed and displacement impact factors are presented in another paper (11).*

Previous Study

In the analysis of potential flow in an ideal fluid of infinite extent, at rest at infinity, through which a body moves in rectilinear motion at a velocity, v, the kinetic energy of the fluid, T_f, is

$$T_f = 1/2\ Cv^2 \tag{1}$$

where C is a constant factor--with dimensions lb-sec^2/ft--whose

----

*Bibliography appended

value depends on the density of the fluid, the size and shape of the body, and the direction of motion. Correspondingly, the kinetic energy of the body, T_b, is

$$T_b = 1/2\ mv^2 \tag{2}$$

where m is the mass of the body (ratio of its weight to the acceleration due to gravity in a vacuum.) The total kinetic energy, T, of the body-fluid system is then

$$T = T_f + T_b = 1/2\ (C + m)v^2 \tag{3}$$

When the body changes velocity or accelerates, the time rate of change in the kinetic energy of the system is then

$$\frac{dT}{dt} = (C + m)v\ \frac{dv}{dt} \tag{4}$$

where t is time. Since the time rate of change in kinetic energy is the power supplied to the system by an external force F acting on the body,

$$Fv = \frac{dT}{dt} = (C + m)v\ \frac{dv}{dt} \tag{5}$$

or

$$F = (C + m)\ \frac{dv}{dt} \tag{6}$$

Thus, the fluid increases the effective mass of a body from m to (C+m). The quantity (C+m) is usually called the virtual mass and C is the added or induced mass.

A more rigorous explanation of this phenomenon may be found in treatises by Lamb (3) and Milne-Thomson (5).

Birkhoff (1) and Stelson (12) have reviewed and discussed the literature on virtual mass. A summary of general information will not be repeated here since only a small part is applicable to a study of the virtual mass of partially submerged bodies.

In 1929 Von Karman (14) published an analysis of the forces on a wedge as it entered water. He assumed that one-half the added mass obtained from a study of potential flow about an immersed lamina would be applicable to the immersing wedge. He neglected the surface wave. Some experimental data supported his analytical results. Mayo (4) Mitwitsky (6), and Pierson (9), have written more on the problem of an immersing wedge for application to seaplane floats during landings.

In 1930 Taylor (13) published results of his studies of two dimensional potential flow about cylinders having two mutually perpendicular axes of symmetry. He analyzed partially submerged bodies by assuming the special conditions at the free surface to be satisfied when an axis of symmetry coincided with the surface. He attempted to estimate the end effect in three dimensional flow by a comparison of other shapes with ellipsoids for which the added mass is known.

Taylor has analyzed the flow around a cylinder of infinite length (two dimensional flow) whose cross-section was formed by two identical circular arcs. For movement perpendicular to the common chord he found the added mass, C, per unit length to be

$$C = \frac{\pi}{3}(n^2 + 2)-2 \left[(\pi - \frac{\pi}{n}) \operatorname{cosec}^2 \frac{\pi}{n} + \cot \frac{\pi}{n}\right] \qquad (7)$$

in a fluid of unit density where $\frac{2\pi}{n}$ is the external angle between tangents to the two arcs at their points of intersection. One-half of this result is applicable to a partially submerged circular cylinder when the free surface corresponds to the common chord. In Table 1 are some of the numerical results presented by Taylor.

TABLE 1

ADDED MASS--CIRCULAR ARCS

n	$\frac{2\pi}{n}$	Radius	Chord length	Thickness	C
1	360°		2	0	3.14 (flat plate)
*6/3 [6/5]	300	2	2	0.536	2.88
4/3	270	1.41	2	0.828	2.82
3/2	240	1.15	2	1.15	2.81
2	180	1	2	2	3.14 (circle)

For a cylinder whose cross-section is a rhombus, Taylor also analyzed the flow for movement in the direction of a diagonal. He found the added mass per unit length, C, in a fluid of unit density to be

$$C = 2n\pi - s^2 \sin 2n\pi \tag{8}$$

where

$$s = \frac{\Gamma(1/2 + n)\,\Gamma(1 - n)}{2\,\Gamma(3/2)} \tag{9}$$

and $n\pi$ is the angle between the diagonal in the direction of motion and the sides. When $n\pi = 45^{\circ}$, the rhombus is a square and the added mass is

$$C = 0.594\, b^2 \tag{10}$$

where b is the length of a diagonal. One-half of this result is applicable to a 90° wedge when the angle bisector is perpendicular to a free surface.

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*Note that n=6/3 should read n=6/5

Riabouchinski (10) analyzed the flow about a cylinder having a rectangular cross-section moving broadside-on. He found the added mass, C, for a cylinder of unit length in a fluid of unit density to be

$$C = \pi \left(\frac{w}{2}\right)^2 \left\{ \frac{\sin^2\alpha}{\left[E - (\cos^2\alpha)K\right]^2} - \frac{4}{\pi}\frac{d}{w} \right\} \quad (11)$$

where α is a parameter such that

$$\frac{d}{w} = \frac{E' - (\sin^2\alpha)K'}{E - (\cos^2\alpha)K} \quad (12)$$

The broadside width is w and the thickness is d. The terms K and E are the complete elliptic integrals of the first and second kind and $\cos\alpha$, $\sin\alpha$, are their respective moduli. The terms K' and E' are corresponding functions of the complementary angle $90^\circ - \alpha$. In Table 2 some of the numerical results of Riabouchinski's analysis are reproduced.

TABLE 2

ADDED MASS--RECTANGLES

α	$\frac{d}{w}$		$\frac{C}{\pi(\frac{w}{2})^2}$	
90°	0		1 (flat plate)	
80°	0.025		1.05	
70	0.111	[0.113]	1.16	[1.15]
60	0.298	[0.301]	1.29	[1.28]
50	0.676	[0.681]	1.42	[1.43]
[45	1.000		1.51]	(square)
40	1.478	[1.468]	1.65	[1.61]

The values in brackets were recomputed as part of the present study according to Riabouchinski's formula. The values for $\alpha = 45^\circ$ were not originally presented. Since $b = 1.41\ w$, the added mass when $\alpha = 45^\circ$ is

$$C = \frac{\pi}{4} w^2\ (1.51) = \frac{\pi}{4} \left(\frac{b}{1.41}\right)^2 (1.51) \tag{13}$$

$$C = 0.594\ b^2 \tag{14}$$

Riabouchinski's value of C for broadside-on movement is identical with the results of Taylor given in equation (10) for edge-on movement of a cylinder with square cross-section is the same in all directions.

Browne, Moullin, and Perkins (2, 7, 8) published the results of their study of vibrations of partially submerged bodies. Setting up forced vibrations by an electromagnet, they measured the added mass of rectangular and triangular prisms for various degrees of submergence. Where comparison is possible, their results are in agreement with the results of the present study.

Stelson (12) has shown that the added mass obtained from studies of two- and three-dimensional potential flow in an ideal fluid of infinite extent is identical to two significant places with the added mass measured in a body of water with finite boundaries. Thus, the validity of applying results from studies of potential flow has been established. However, to obtain the added mass by a study of potential flow around a ponton-shaped body would be extremely difficult if not impossible because of the complex shape and because of a free surface. Since a simple, accurate experimental method of determining added mass had been previously developed at Carnegie Institute of Technology (12), the added mass of the model pontons was measured experimentally so that the study of the behavior of floating bridges could proceed.

EXPERIMENTS

Method

The equivalent added weight was determined by measuring the change in natural fundamental frequency of a freely vibrating steel beam which supported the partially submerged pontons. The relationships between the weight attached at the center and the frequency* of the beams were measured. The water around a ponton attached to the beam would then have the same effect on the beam frequency as would a certain attached weight in air. The equivalent added weight due to the water is then equal to the attached weight in air that causes the same change in beam frequency since their effects are the same.

Apparatus

The tests were made in three long, water-tight flumes. One flume was 18 in. wide having glass walls 0.5 in. thick and a concrete bottom. The second was 36 in. wide having walls and bottom of 3/16-in. steel plate stiffened by additional heavy steel members. The third flume was 58 in. wide having walls and bottom of concrete. The pontons were always tested with their longitudinal axes in the center of the flume parallel to the walls.

Fig 1 shows a sketch of the ponton attached to a beam in the 36-in. flume. As shown in the sketch, the ends of the beam were attached to the supports with plate fulcrums. Fig 2 shows a photograph of a ponton mounted in the 58-in.flume.

Fig. 3 shows the relationships between frequency for the vertical vibration of the test beams and the added weight in air attached to the center of the beams.

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*Hereafter in this report the term frequency will denote the natural fundamental frequency for the free vibration of the beam-body-water system.

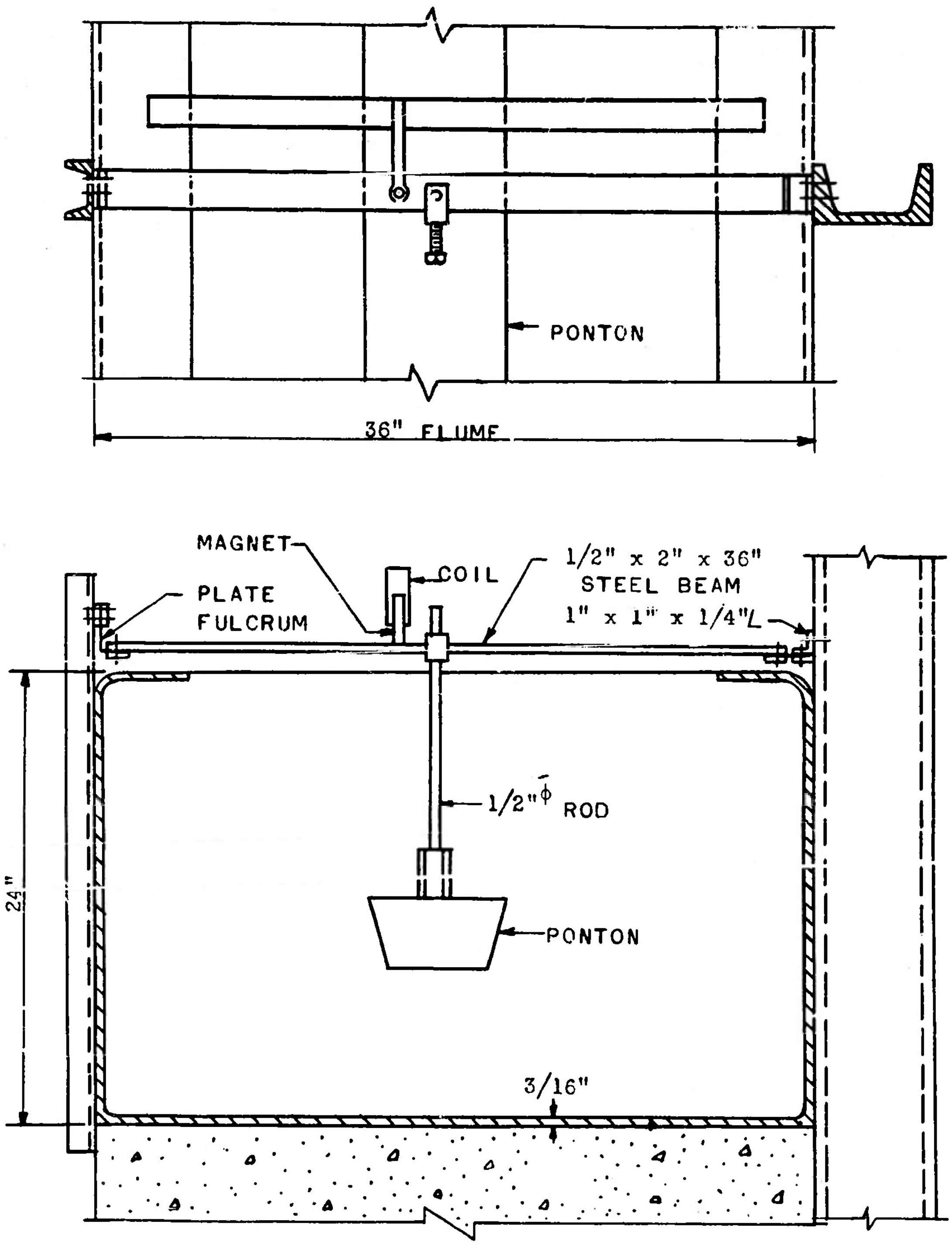

FIG. I THE APPARATUS IN 36-INCH FLUME

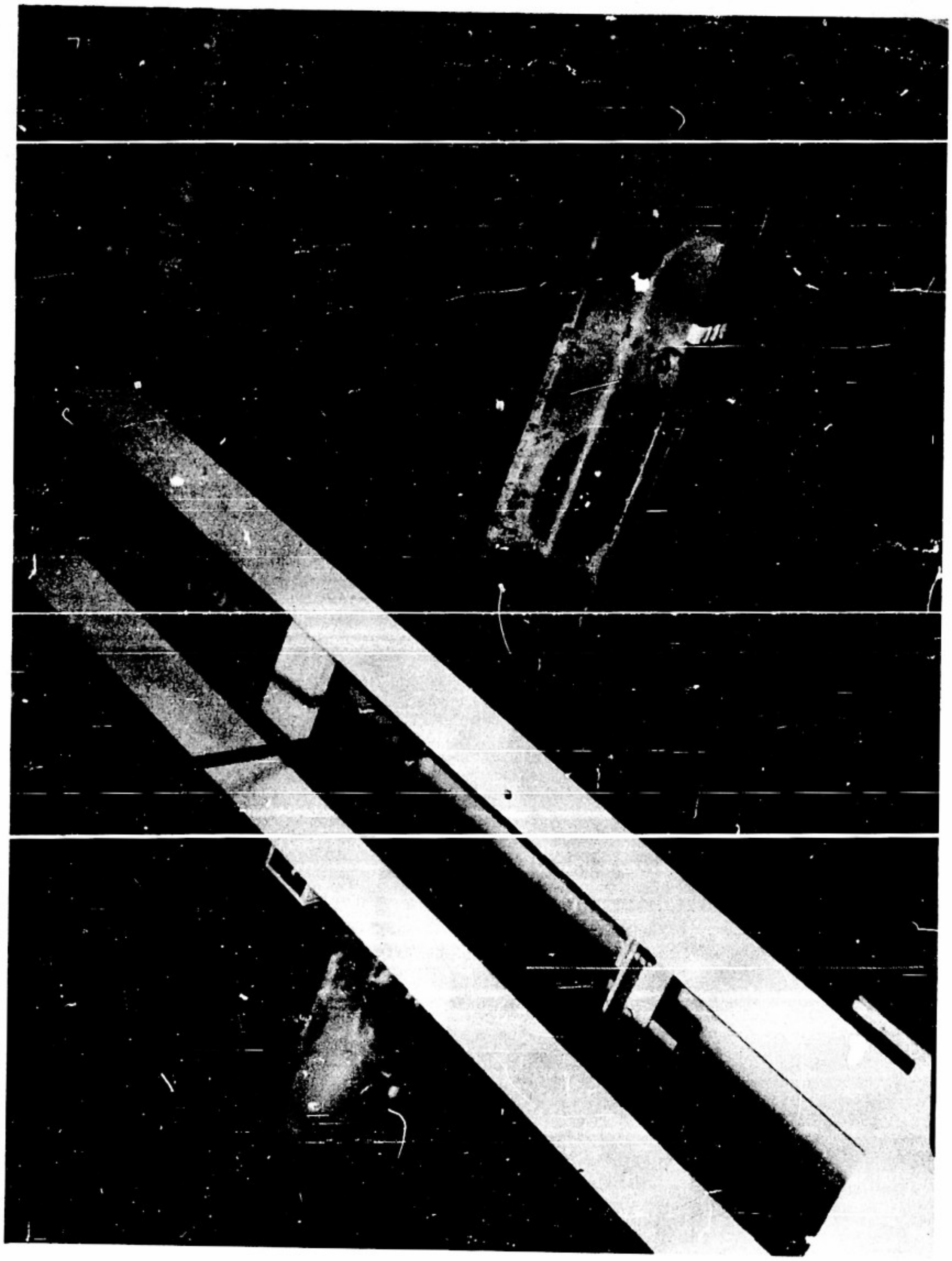

FIG. 2 PHOTOGRAPH OF THE APPARATUS IN THE 58-IN. FLUME.

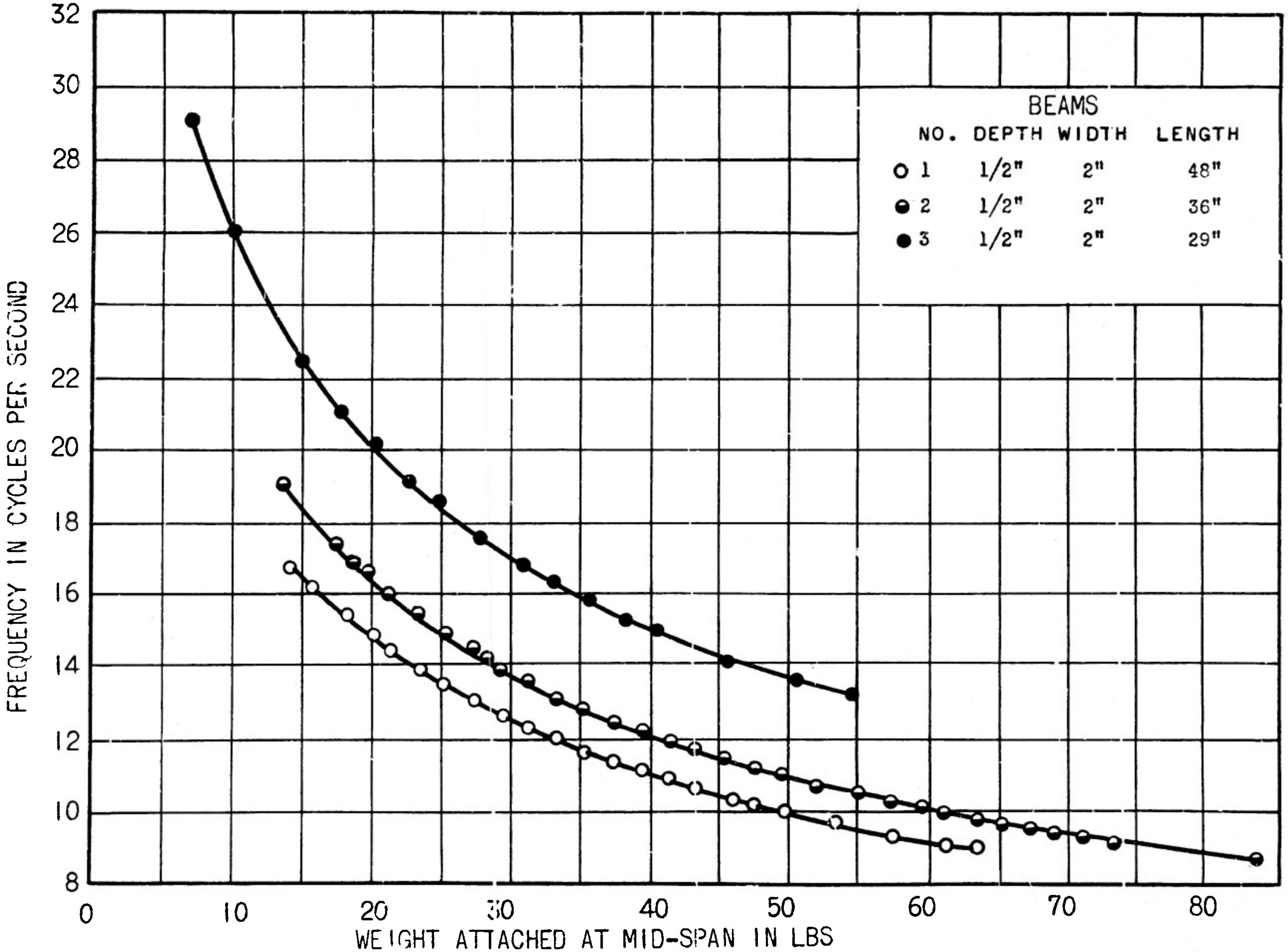

FIG. 3 BEAM CALIBRATION. RELATIONSHIP BETWEEN NATURAL FUNDAMENTAL FREQUENCY OF BEAM AND WEIGHT ATTACHED AT MID-SPAN

The details of the model pontons are shown in Figs 4 and 5. The ponton with raked bow and stern is a one-twelfth scale model of an M-4 ponton borrowed from the Department of Military Science and Tactics, Carnegie Institute of Technology. The pontons with semi-circular, triangular, and rectangular cross-sections were made of smooth transparent plastic 1/4-in. thick. The ends were water-proofed wood. The pontons other than the model M-4 were 4 ft in length when first tested. The lengths of the triangular and rectangular pontons were reduced to 2 ft and the added mass was measured again.

To measure the frequency of the beams, a small electromagnet was fastened to the beams near their centers. A 1 1/2- volt dry cell battery supplied a steady current to the magnet windings. A coil with rigid, independent support was placed around the magnet. As the beam and magnet vibrated, a signal proportional to the vertical velocity of the beam was excited in the coil. The signal was amplified about 100,000 times and recorded on an oscillograph having a chart speed of 12.5 cm per sec. The frequency was obtained by measuring the chart length of 30 to 50 cycles of signal. The damping in the water-ponton-beam system usually did not reduce the vibration amplitude by one-half in 30 cycles. The frequency was measured approximately to $\pm$ 0.03 cycles per sec.

Procedure

The equivalent added weight was determined in the following way:

1. The ponton was submerged to the desired depth and attached to the supporting beam.

2. The elevations of the water surface at both ends of the ponton were measured to $\pm$ 0.0005 ft. If the two ends did not have the same elevation, the apparatus was adjusted to make them the same.

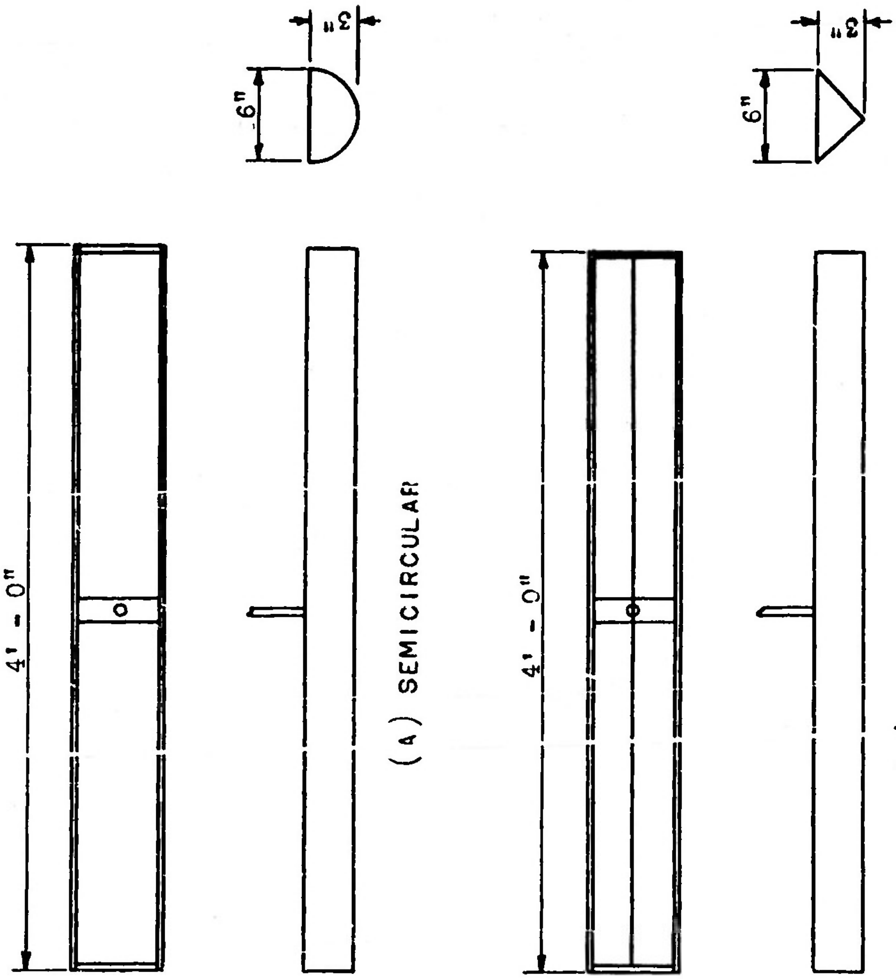

FIG. 4 MODEL PONTONS

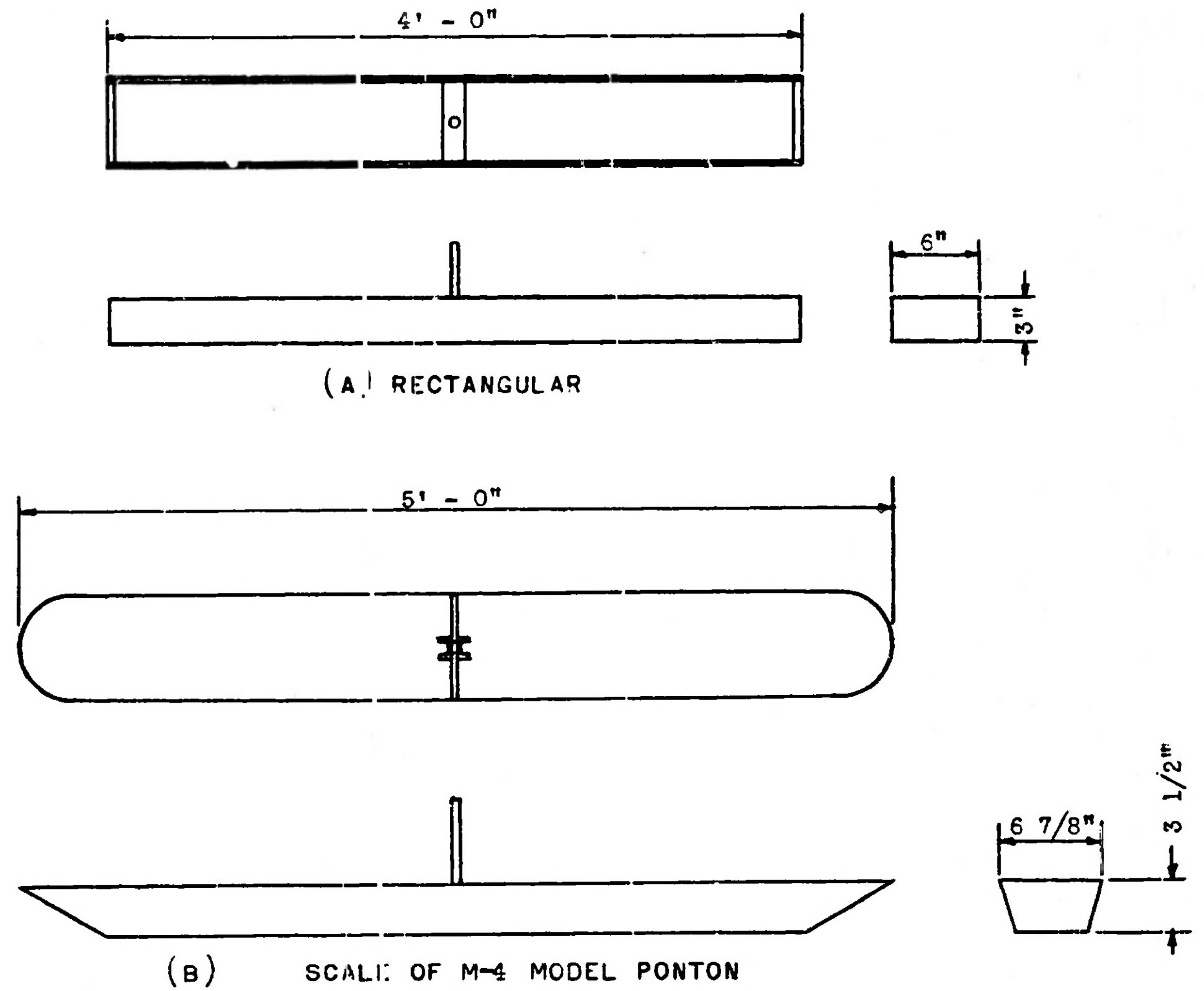

FIG. 5 MODEL PONTONS

3. When the water was quiet, the center of the beam was lightly tapped with a finger. After the first vibration had died away, the beam was tapped a second time.

4. The vertical velocity of the center of the beam was recorded, and the frequency of the vibration was determined. The beam was tapped twice as a check. The difference in the two measurements of frequency was never greater than 0.05 cycles per sec when the apparatus was operating correctly.

5. The equivalent attached weight in air that gave the same frequency was determined from the relationship show in Fig 3.

6. The actual weight of the ponton was subtracted from the equivalent attached weight to give the equivalent added weight due to the water.

7. A four- or six-pound weight was attached to the beam and the test was repeated. The results of the first test were checked in this way for possible mistakes in measurement or computation.

RESULTS AND ANALYSIS

Fig 6 shows the relationship between the frequency of vibration of beam No. 2 and the depth of submergence of the attached model M-4 ponton. The tests were made in the 36-in. flume. Clearance between the bottom of the ponton and the bottom of the flume is shown as a parameter. Fig 7 shows the relationship between the equivalent added weight and the submergence of the M-4 model ponton. The equivalent added weight was obtained from the data shown in Figs. 3 and 6. Fig 8 shows the measured relationship between the submergence of the M-4 model ponton and the displaced weight of water. Fig 9 shows the relationship between the ratio of equivalent added weight to displaced weight of water and the submergence of the ponton.

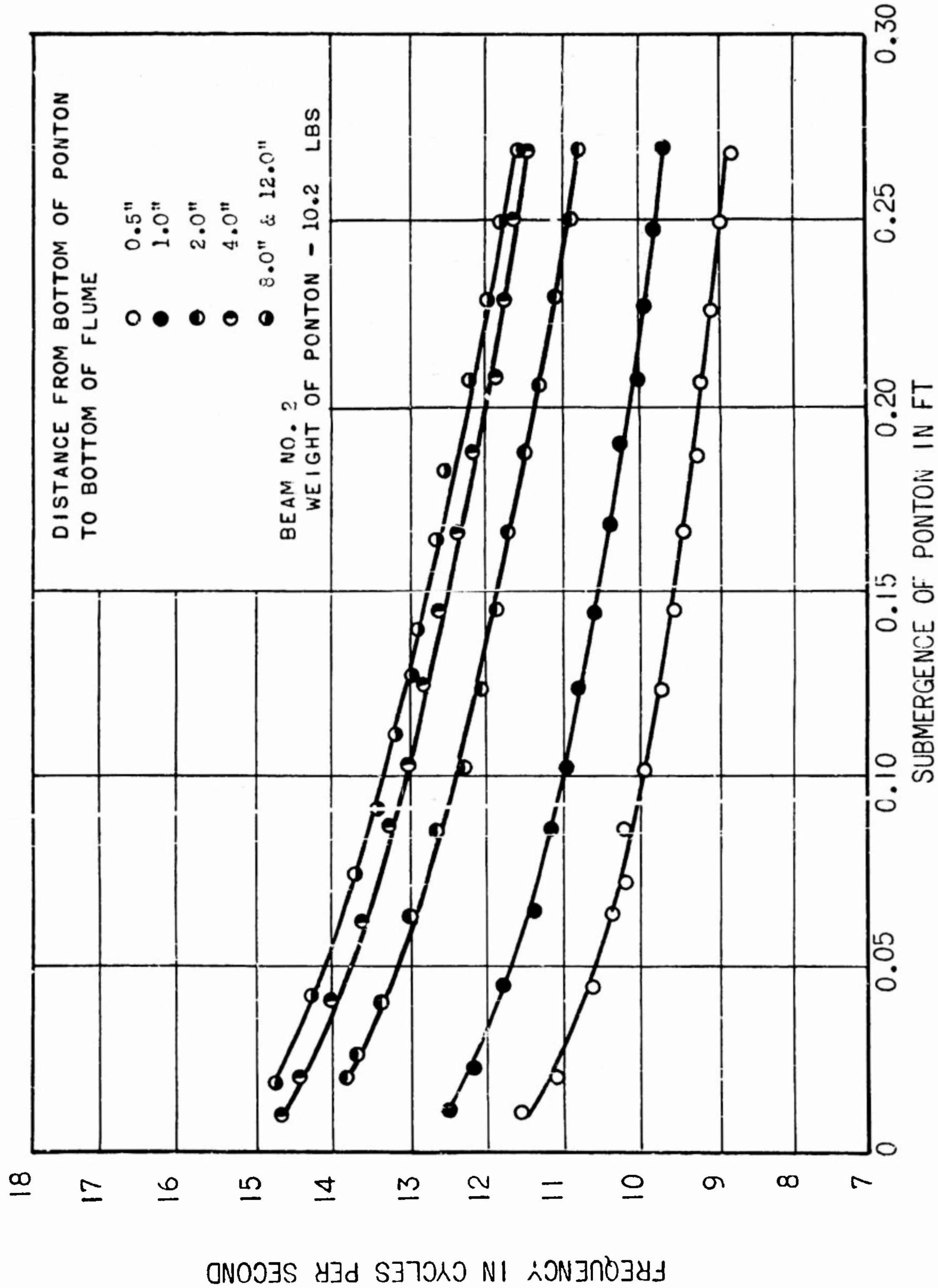

FIG. 6 RELATIONSHIP BETWEEN BEAM FREQUENCY AND DEPTH OF SUBMERGENCE FOR M - 4 MODEL PONTON

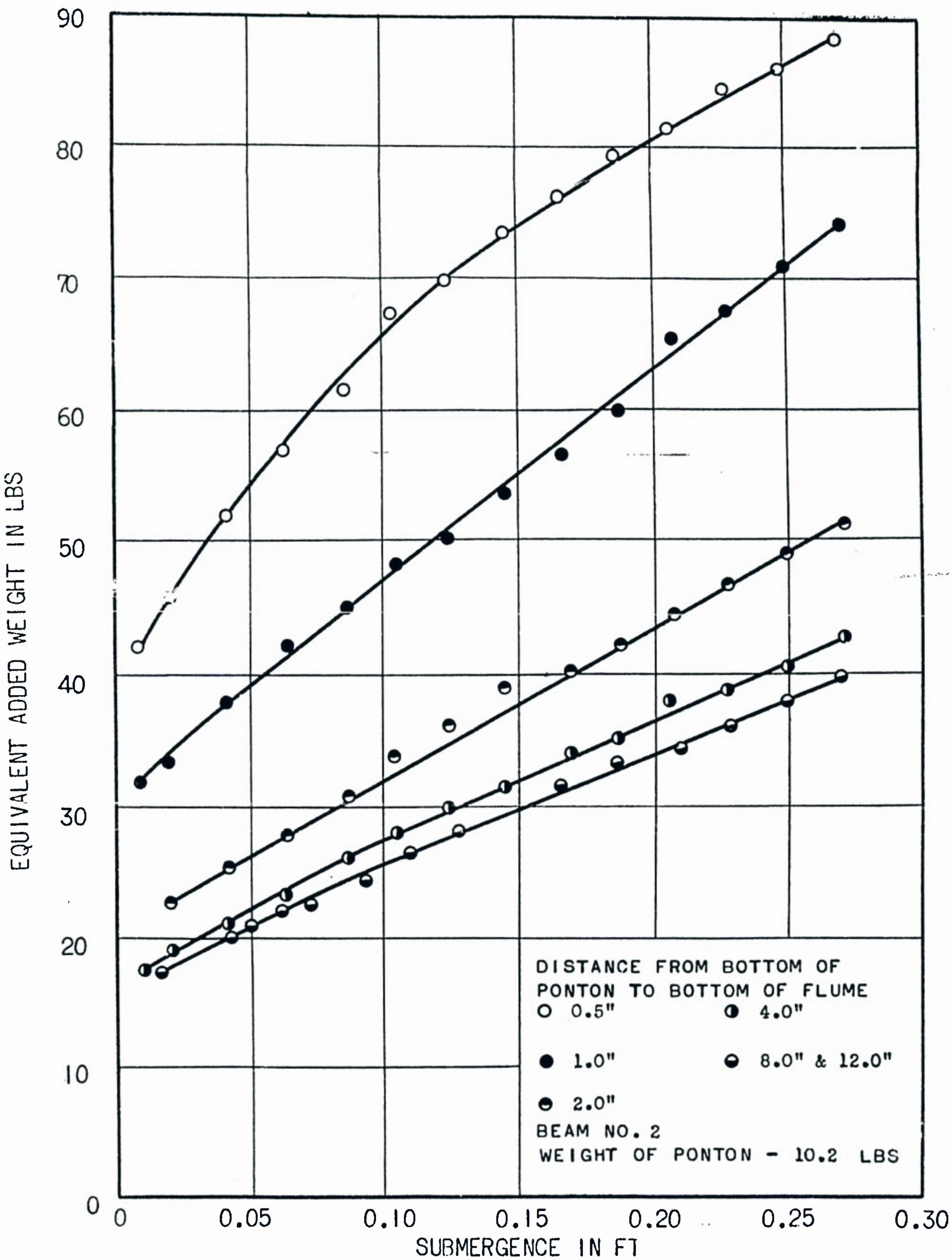

FIG. 7 RELATIONSHIP BETWEEN EQUIVALENT ADDED WEIGHT, SUBMERGENCE, AND BOTTOM CLEARANCE FOR M-4 MODEL PONTON

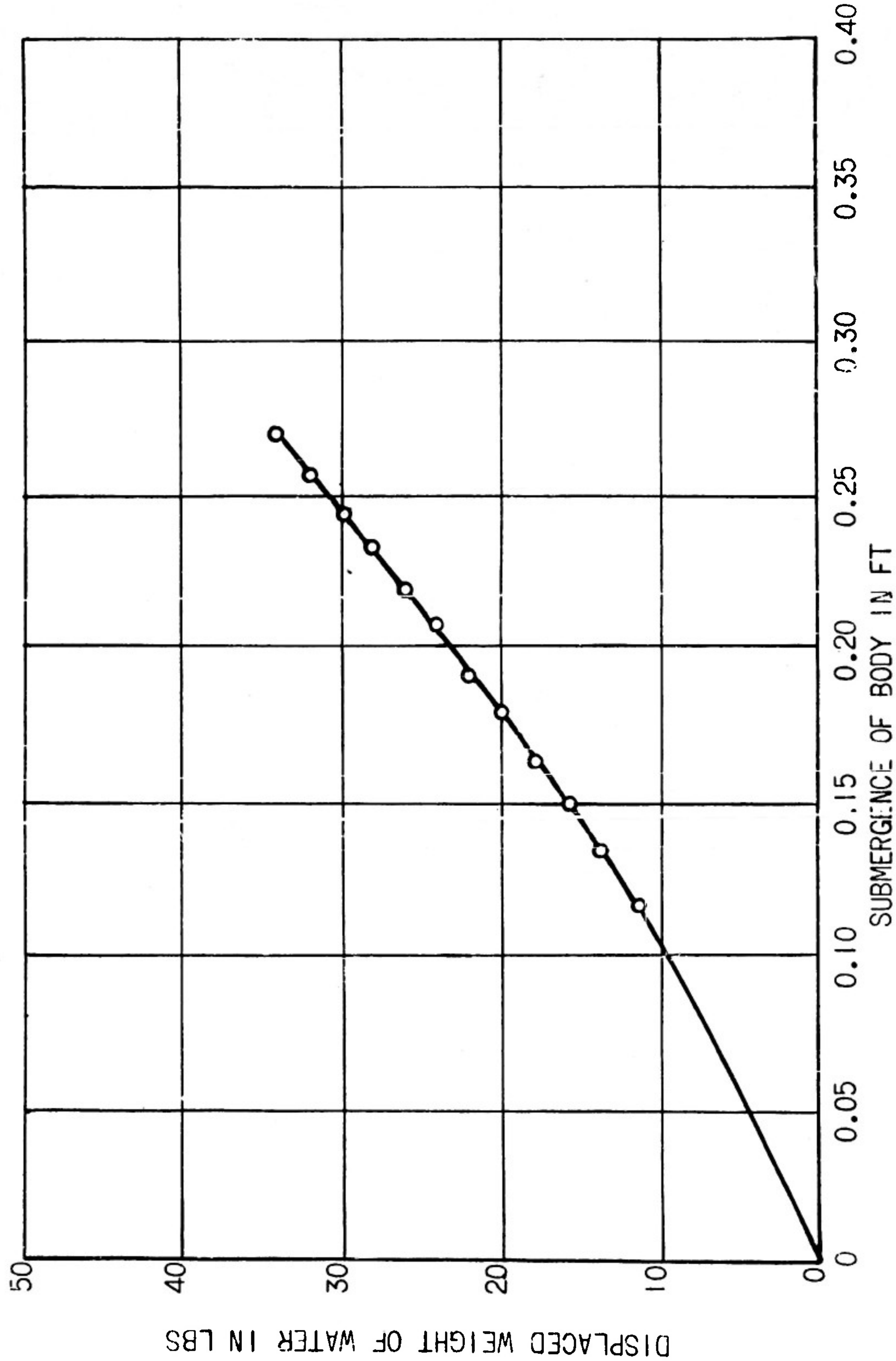

FIG.8 RELATIONSHIP BETWEEN DISPLACED WEIGHT OF WATER AND THE DEPTH OF SUBMERGENCE OF M-4 MODEL PONTON

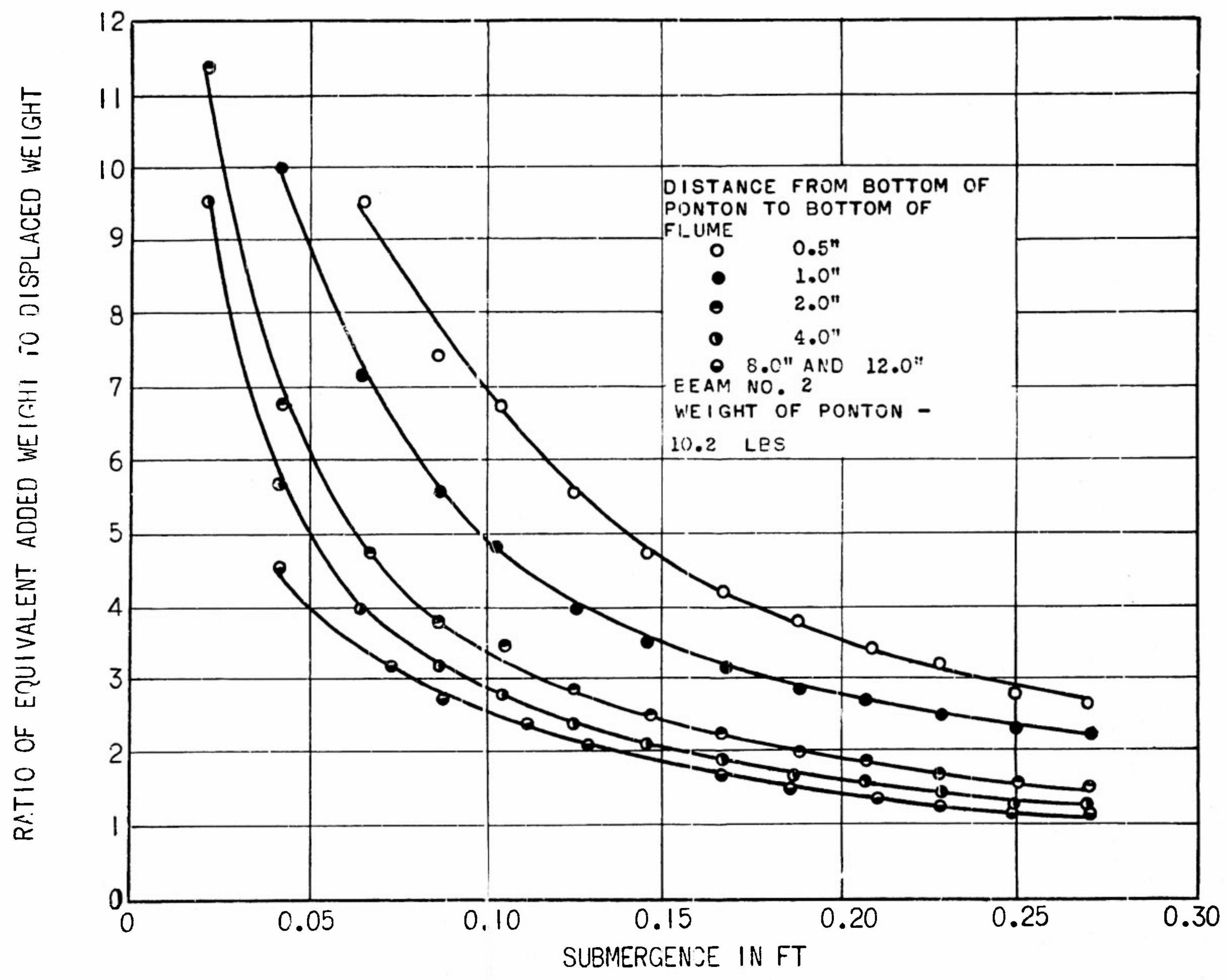

FIG. 9 RELATIONSHIP BETWEEN RATIO OF EQUIVALENT ADDED WEIGHT TO DISPLACED WEIGHT AND THE DEPTH OF SUBMERGENCE FOR M-4 MODEL PONTON

The information shown in Fig 9 was obtained from Figs 7 and 8.

The final results as shown in Fig 9 are in a convenient form for use in the analysis of the dynamic behavior of a ponton. Note that the equivalent added weight may be several times the displaced weight of water. When the bottom clearance is reduced below 4 inches, the equivalent added weight is greatly increased. For bottom clearance greater than 8 inches the effect of the bottom was negligible. Note that the data for 8- and 12- inch clearance are the same.

Flume Size

Fig 10 shows the effect of container size on the equivalent added weight. In the 18- and 36-in. flumes the distance from the bottom of the ponton to the bottom of the flume was 12 in. In the 58-in flume the bottom clearance was 36 in. These measurements show that the side clearance has a sizable effect on the equivalent added weight.

Ponton Rigidity

The amplitude of vibration of the beam and test body is very small--less than 0.01 inch as measured with a visual amplitude gage. Because the movement is so small, the rigidity of the ponton is important. If the ponton were too flexible, the measured equivalent added weight would be erroneously large. The equivalent added weight of the original model pontons as shown in Figs 4 and 5 were determined; the pontons were reinforced with a 2 x 6 in. piece of fir wood; and the equivalent added weight was determined again. In Fig 11 the results of the tests on the stiffened and unstiffened rectangular pontons are shown. The pontons as originally constructed were too flexible. Hence, all the tests described hereafter in this report refer to the stiffened pontons.

The stiffness is expressed as the product $\frac{EI}{L^3}$ where E is the modulus of elasticity; I, the moment of inertia about a horizontal axis through the centroid; and L, the half-length of the ponton. The modulus of elasticity of lucite was taken as 400,000 lbs per sq in. and of fir wood, 1,800,000 lbs per sq in. A stiffness greater than approximately 1000 lbs/in. is required to remove measurable error due to flexibility. Note in Fig 11 that increasing the stiffness from 580 to 24,000 lbs/in. reduced the equivalent added weight by only 6 per cent.

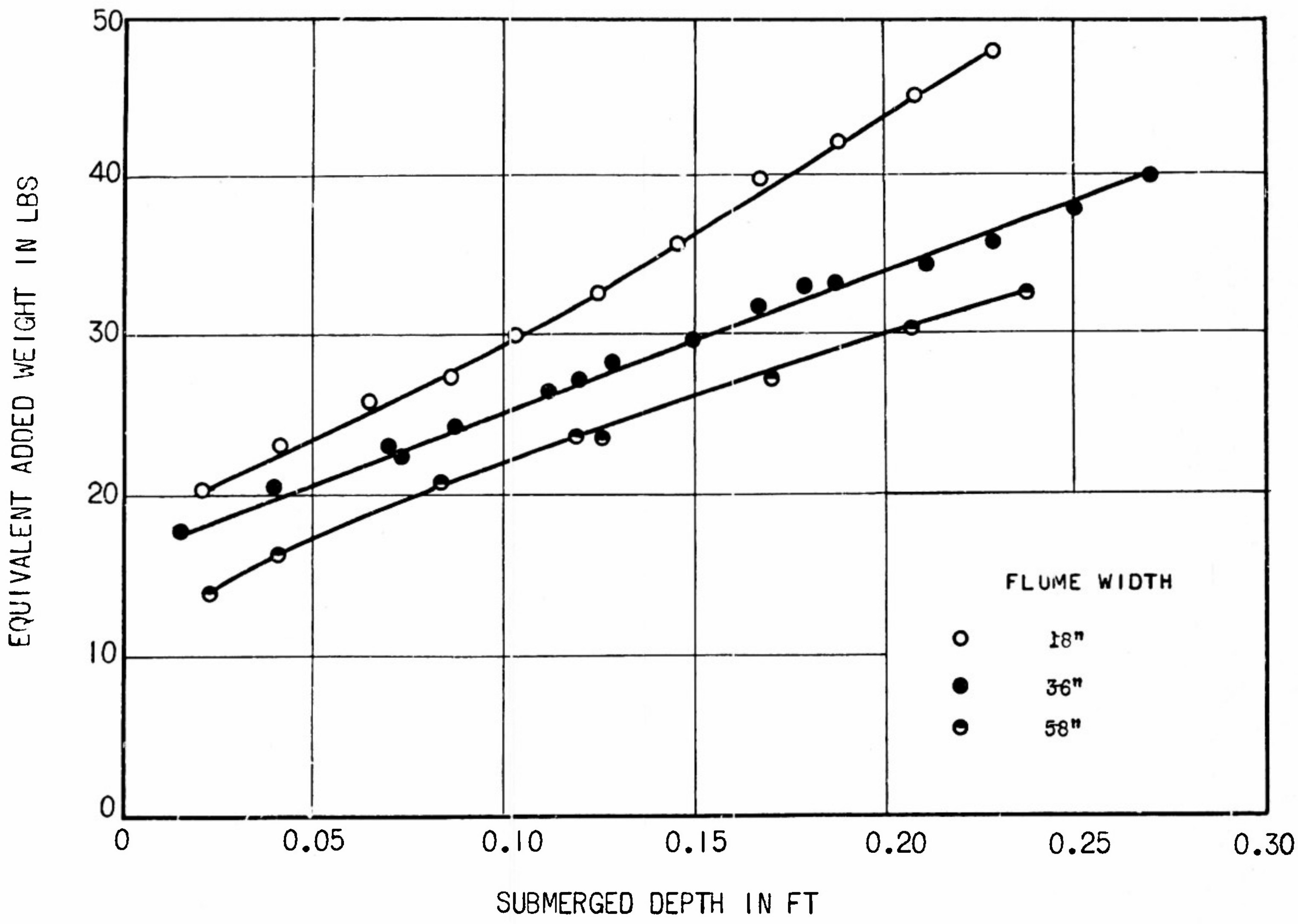

FIG. 10 EQUIVALENT ADDED WEIGHT OF MODEL M-4 PONTON IN THREE FLUMES

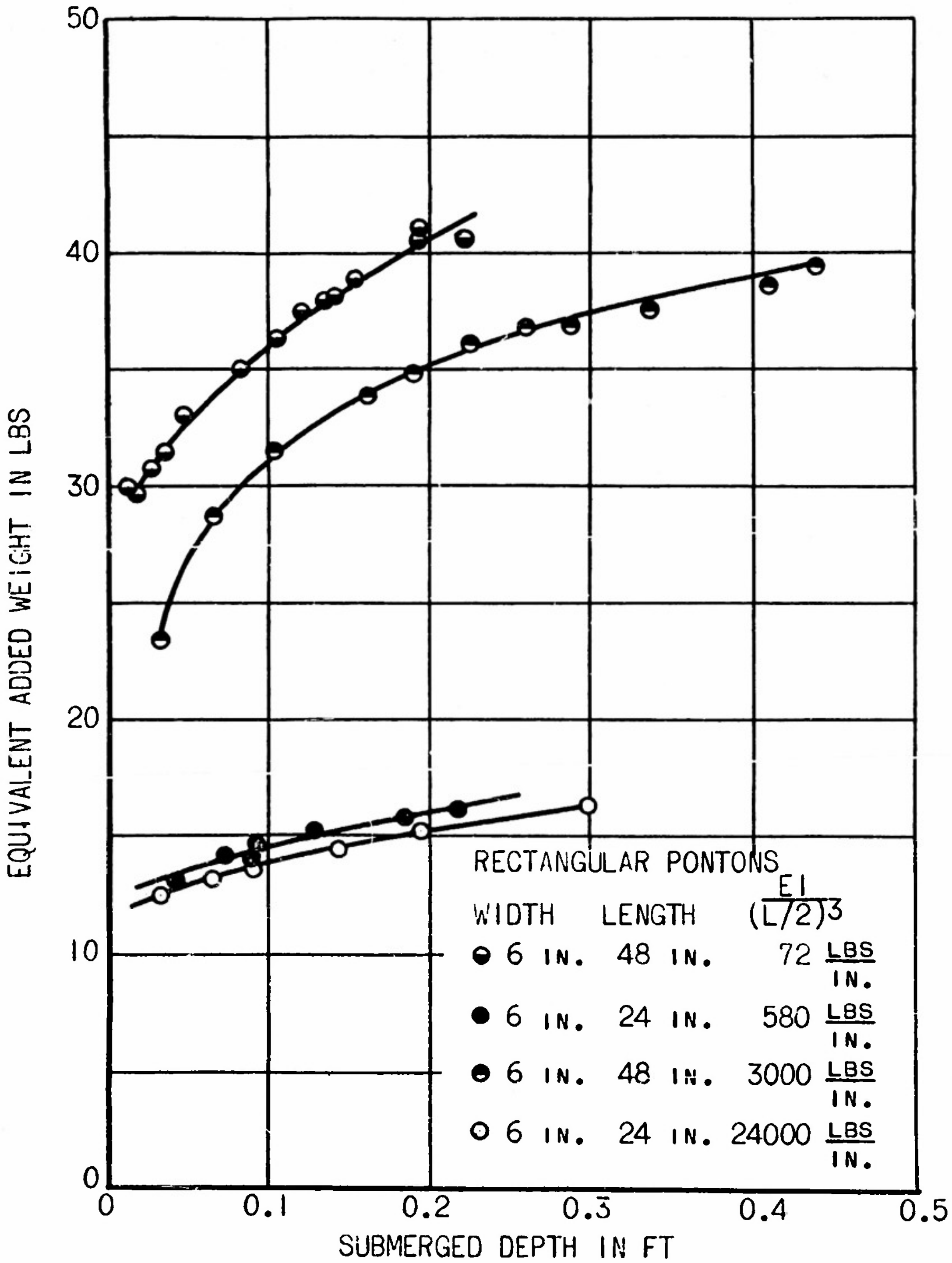

FIG. 11 EFFECT OF PONTON STIFFNESS ON THE MEASURED EQUIVALENT ADDED WEIGHT

Rectangular Pontons

Fig 12 shows the relationship between the frequency of beam No. 3 and $2D/B$, the ratio of twice the submerged depth to the width, for attached rectangular pontons 2 and 4 ft long. In Fig 13 the equivalent added weight is shown as a function of the ratio of $2D/B$. The experimental data of Browne, Moullin, and Perkins (2) are shown as corrected for different widths and lengths to a length of 48 in. and a width of 6 in. The length was corrected in direct ratio. For example, the data for the ponton 6 in. wide and 54 in. long were multiplied by the factor 48/54. The width correction was as the square. For example, data for the ponton 9 in. wide and 108 in. long were multiplied by $6^2/9^2 = 36/81$ to correct for width and 48/108 to correct for length.

The width correction is consistent with theory. The length correction, however, neglects the reduction in equivalent added weight caused by the finite length.

To illustrate, suppose that the reduction in equivalent added weight due to the finite length of the 24- and 48- in. pontons 6 in. wide is the same. Let this value be <u>B</u>. Let <u>W</u> be the equivalent added weight of a 24-in. section of a very long ponton 6 in. wide. The equivalent added weight of the finite 24 in. ponton is then $W - B$ and of the 48 in. ponton $2W - B$. Applying a direct length correction ratio, however, one has $(W - B)(48/24) = 2W - 2B$. Thus, twice the equivalent added weight of a 24 in. ponton should be an end effect less than the equivalent added weight of the 48 in. ponton.

In Fig 13 all data are less than the theoretical values for infinite length because of end effect due to finite length. The longer cylinders should more closely approach the curve from theory. In this respect the

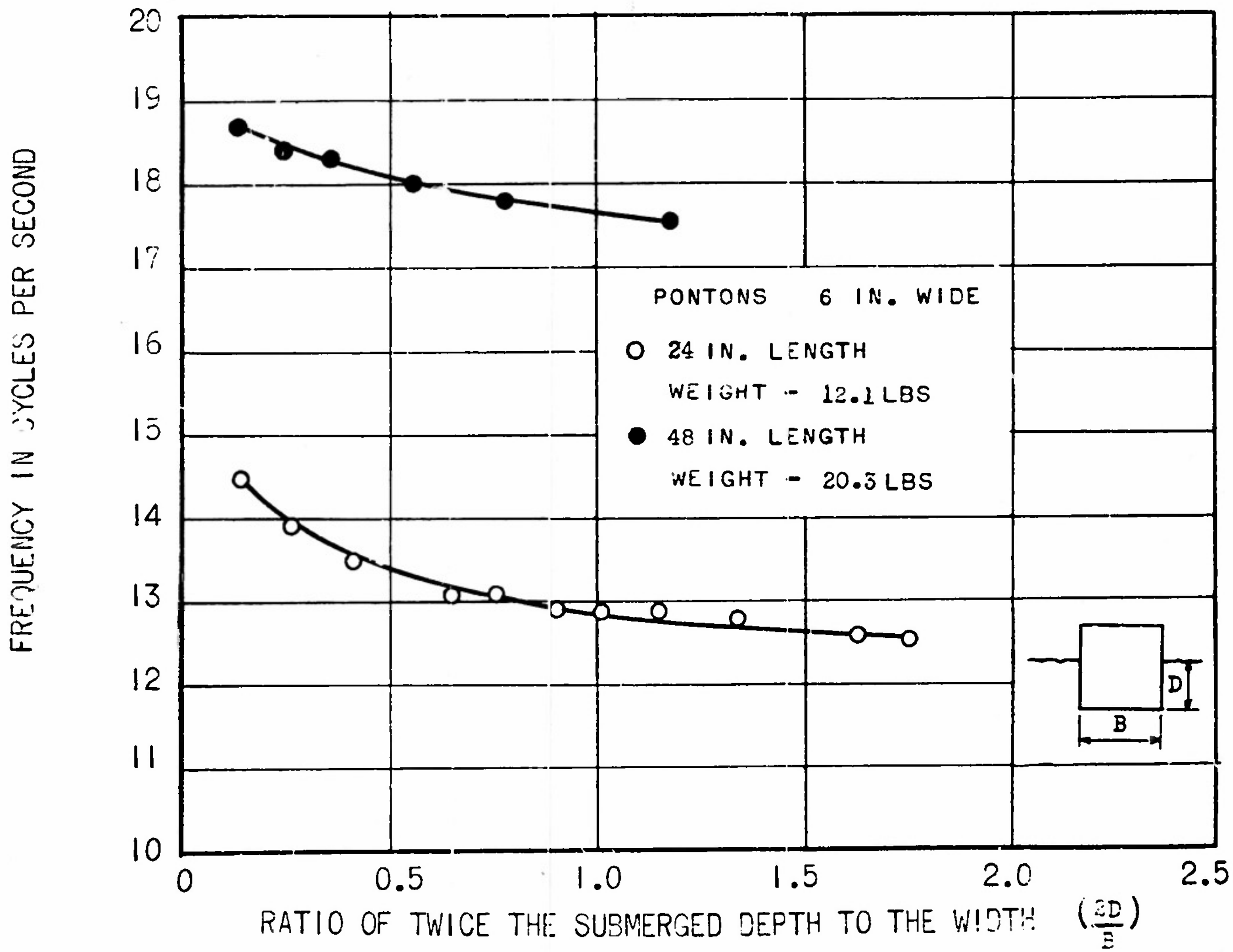

FIG. 12 NATURAL FREQUENCY AND SUBMERGENCE FOR RECTANGULAR PONTONS

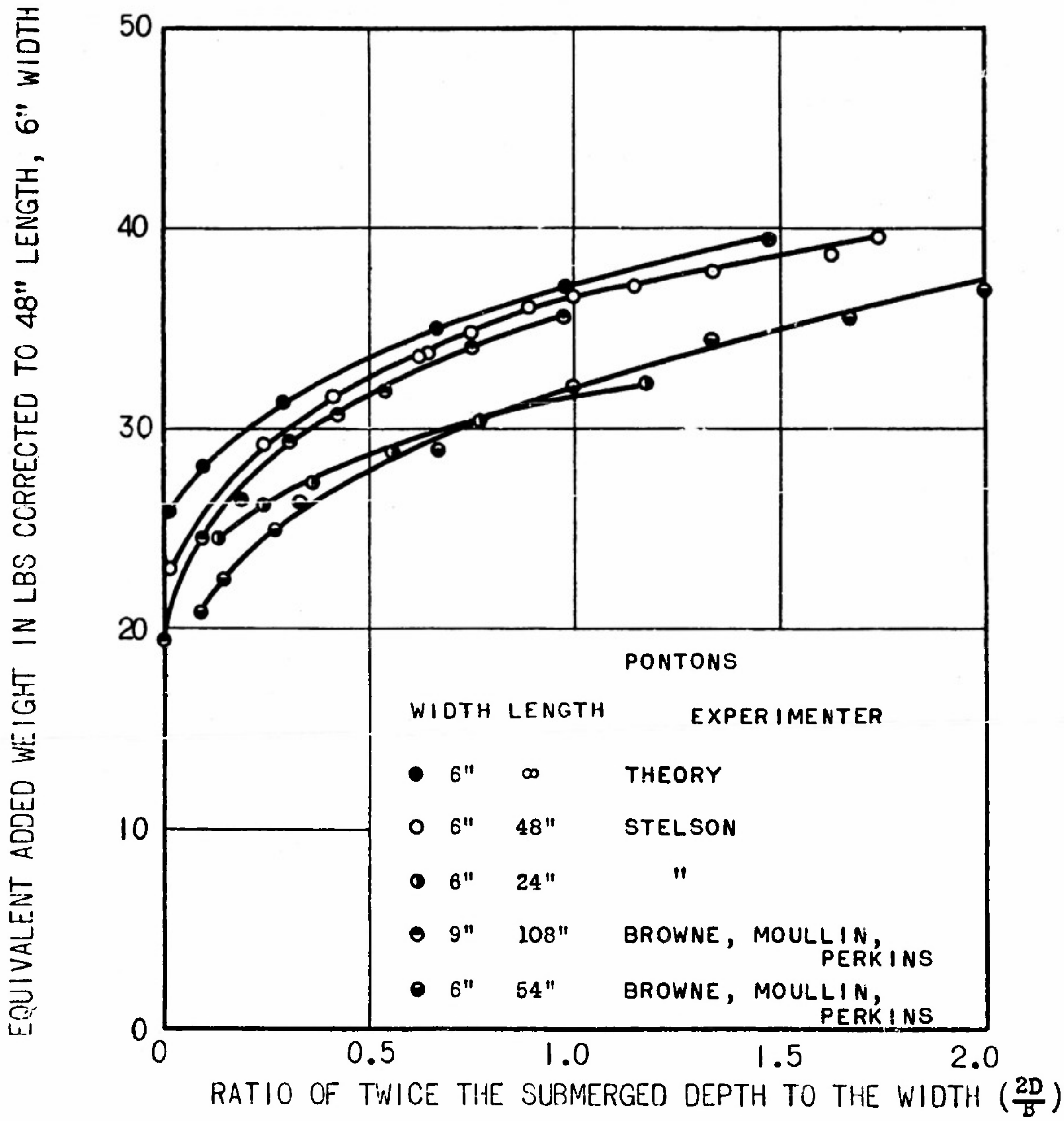

FIG. 13 EQUIVALENT ADDED WEIGHT AND SUBMERGENCE FOR RECTANGULAR PONTONS

data are consistent except for that on the 54-in. ponton by Browne, Moullin, and Perkins. The theoretical relationship is that of Riabouchinski (10).

Fig 14 shows the relationship between the ratio of equivalent added weight to displaced weight and ($^{2D}/B$), the ratio of twice the submerged depth to the width for the same rectangular pontons.

Triangular Pontons

Fig. 15 shows the relationship between the frequency of beam No. 3 and the square of the submerged depth of pontons 24 and 48 in. long having triangular cross-sections. As shown in Fig 4(b) the section is symmetrical about a vertical centerline and has a 90° internal angle at the bottom.

Fig. 16 shows the relationship between the equivalent added weight of the triangular pontons and the square of the submerged depth. The data are corrected to 48 in. length in direct ratio. The data of Browne, Moullin, and Perkins for pontons 54 and 108 in. long are shown. The straight line is the relationship obtained from theory for cylinders of infinite length with square cross-sections moving edge-on. Taylor (13) found the added mass per unit length of such a body in a fluid of unit density to be

$$C = 0.594\ b^2 \qquad (10)$$

where b is the length of diagonal. The equivalent added weight, W, of a 90° V-bottomed ponton 48 in. long in water is then

$$\begin{aligned} W &= \frac{1}{2}\,(0.594)\left(\frac{2D}{12}\right)^2 (4)\,(62.4) \\ &= 2.06\ D^2 \qquad (15) \end{aligned}$$

where W is in lbs, D is in in., 4 is the length in ft and 62.4 is the density of water in lbs per cu ft. The experimental data for the

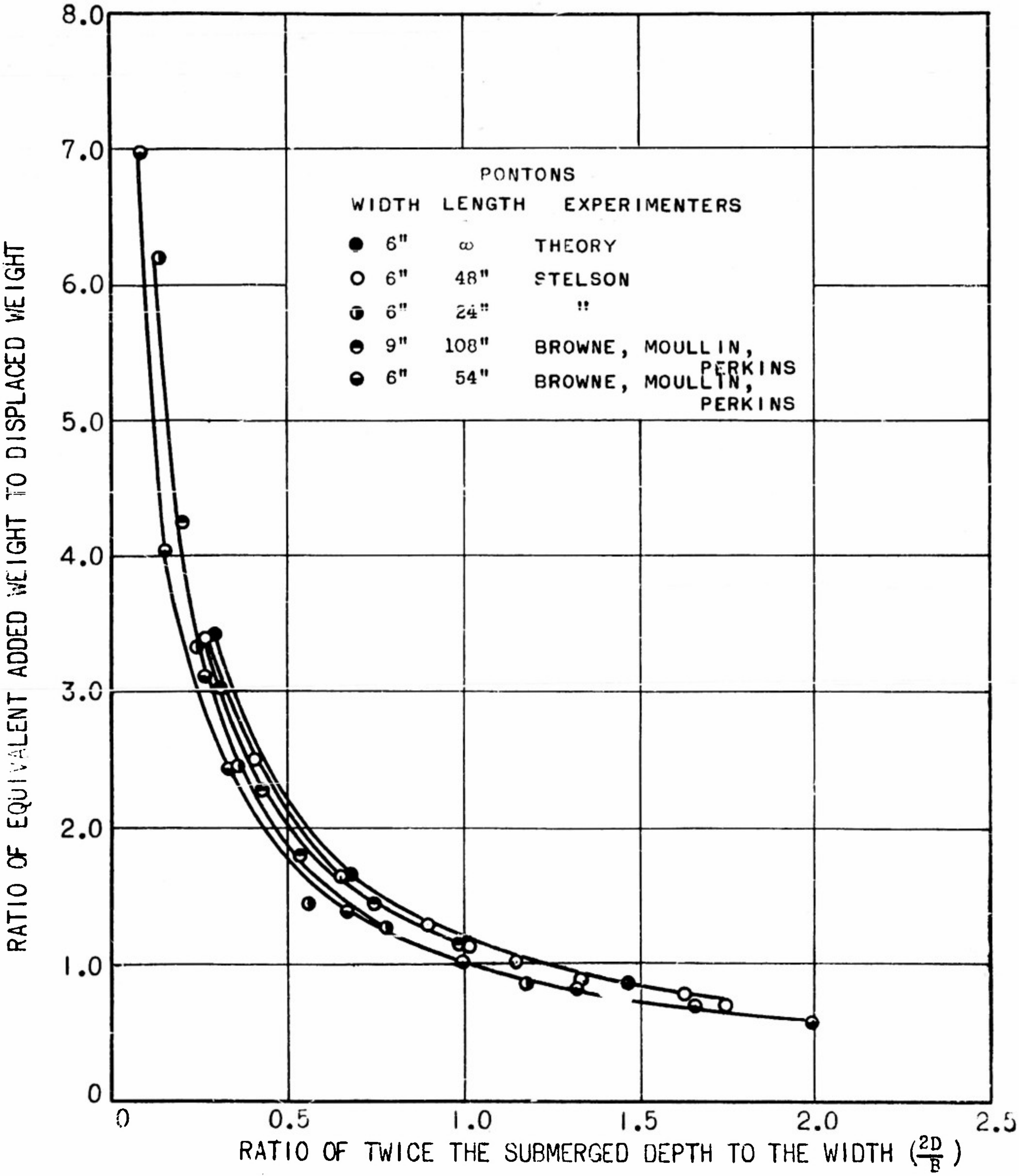

FIG. 14 RATIO OF EQUIVALENT ADDED WEIGHT TO DISPLACED WEIGHT AND SUBMERGENCE FOR RECTANGULAR PONTONS

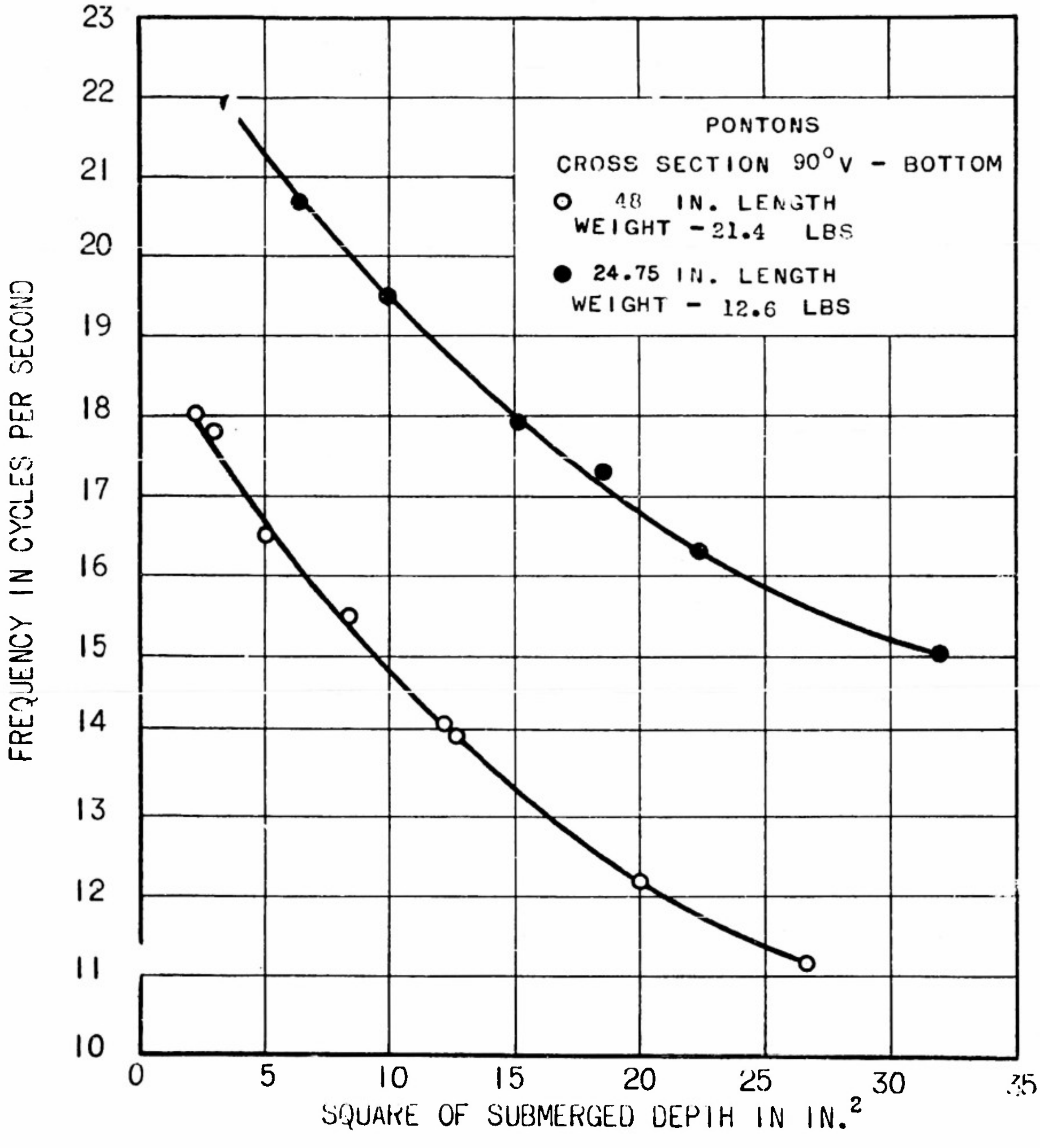

FIG. 15 FREQUENCY AND SQUARE OF THE SUBMERGED DEPTH FOR TRIANGULAR PONTONS

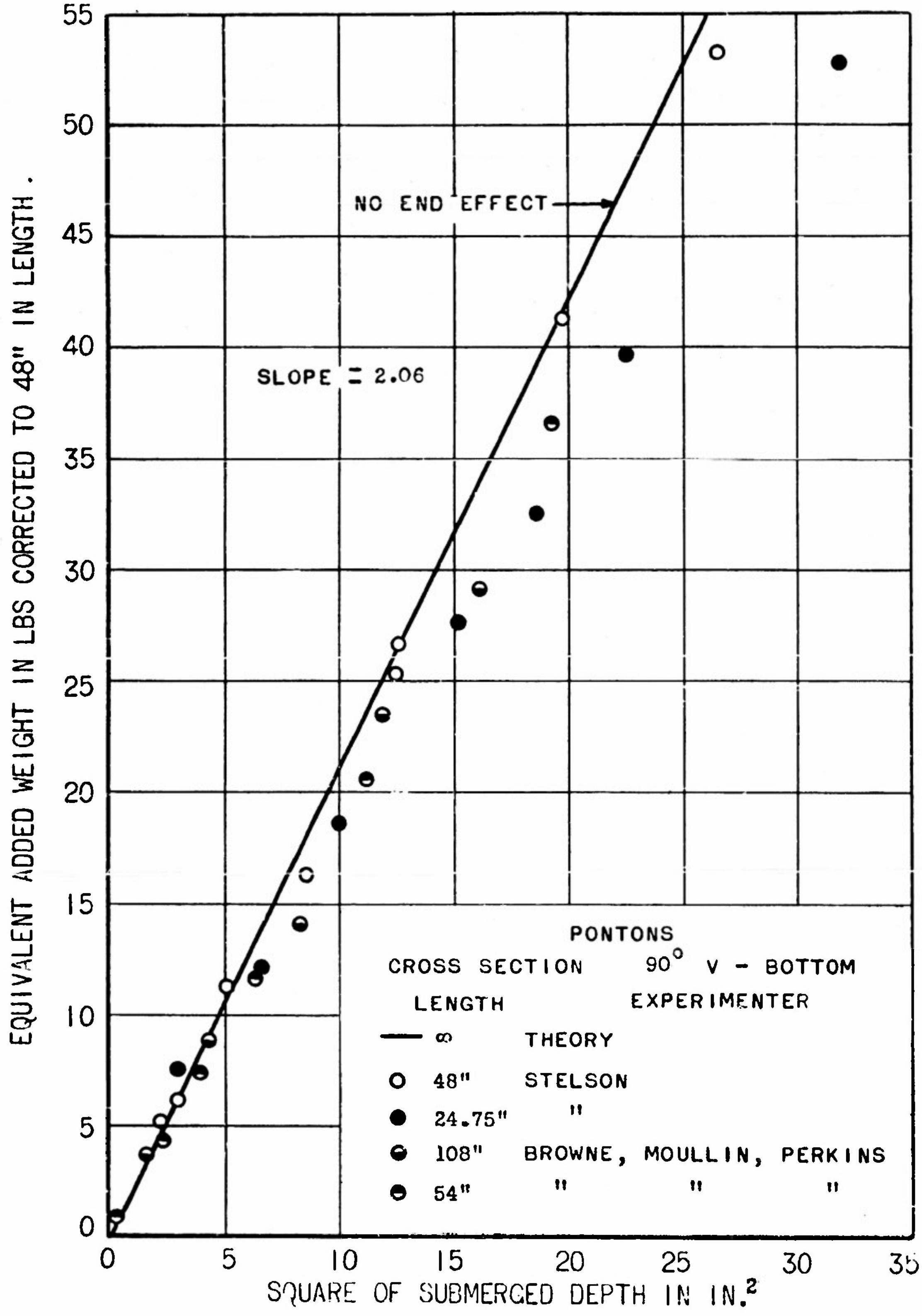

FIG. 16 EQUIVALENT ADDED WEIGHT AND SQUARE OF THE SUBMERGED DEPTH FOR TRIANGULAR PONTONS

longer pontons are very close to this theoretical relationship as shown in Fig 16.

The weight, W', displaced by 90° V-bottomed ponton 48 in. long with square ends is

$$W' = \frac{1}{2} \left(\frac{D}{12}\right) \left(\frac{2D}{12}\right) (62.4)\ (4)$$

$$= 1.73\ D^2 \qquad (16)$$

The ratio of equivalent added weight to displaced weight for very long 90° V-bottomed ponton would then be 2.06/1.73 = 1.19. Fig 17 shows the measured ratio of equivalent added weight to displaced weight as a function of submergence. The data for the longer pontons are very close to the 1.19 value. The shorter pontons have lower ratios as would be expected because of the proportionally greater end effect.

Semi-circular Ponton

Fig 18 shows the measured relationship between the frequency of beam No. 3 and the depth of submergence of a semi-circular ponton 6 in. in diameter. Fig 19 shows the equivalent added weight as a function of the depth of submergence. The theoretical relationship between equivalent added weight and depth of submergence for a 48 in. section of a very long ponton is also shown in Fig 19. These values were obtained from Taylor (13).

In Fig 20 the ratio of equivalent added weight to displaced weight is shown as a function of the submerged depth for the semi-circular ponton.

Vibration and Added Mass

The added mass has a great influence on the resonant vibrational frequencies of a ponton. The determination of the resonant frequencies is important in the safe operation of a floating bridge. Vehicle

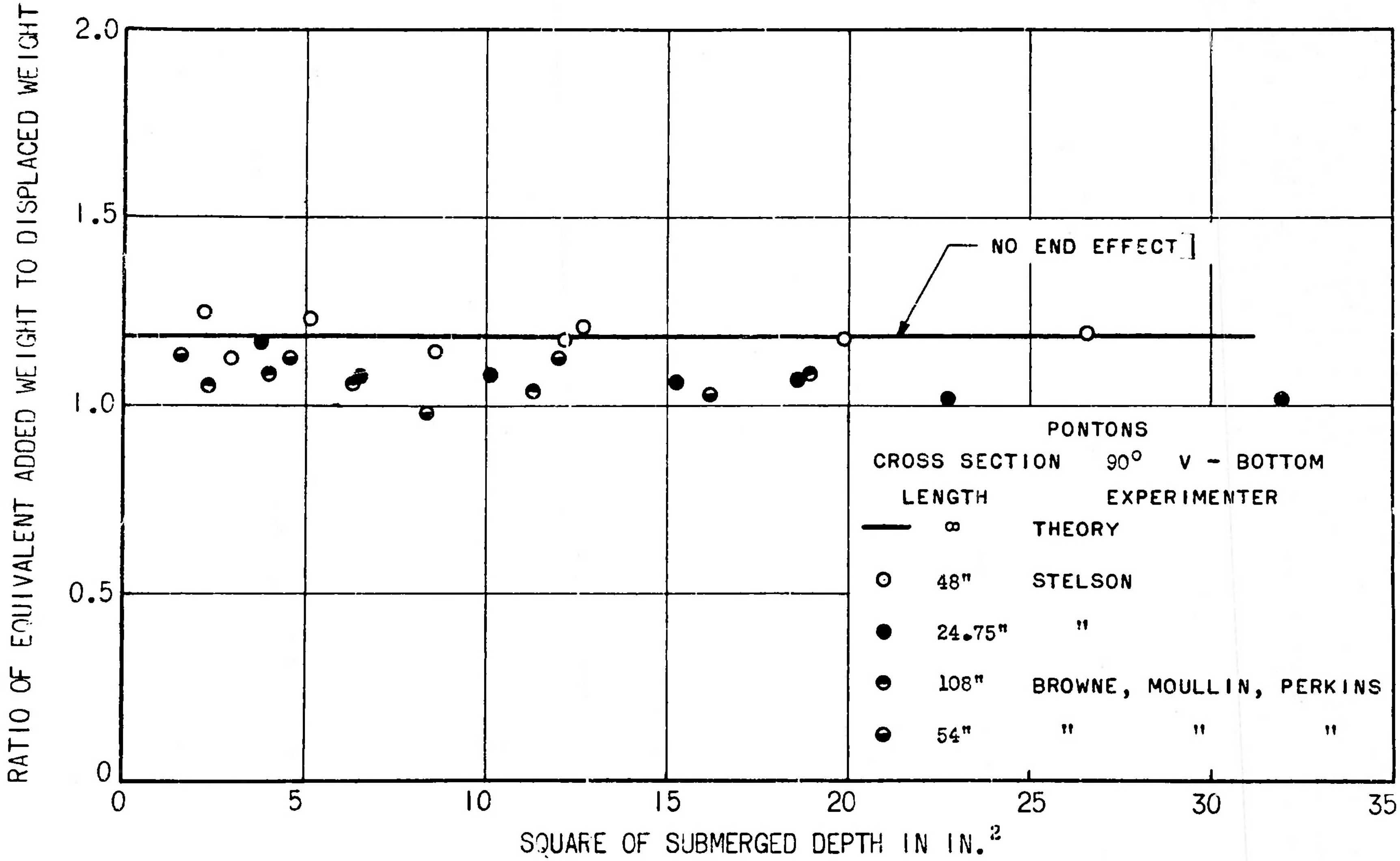

FIG. 17 RATIO OF EQUIVALENT ADDED WEIGHT TO DISPLACED WEIGHT AND THE SQUARE OF THE SUBMERGED DEPTH FOR TRIANGULAR PONTONS

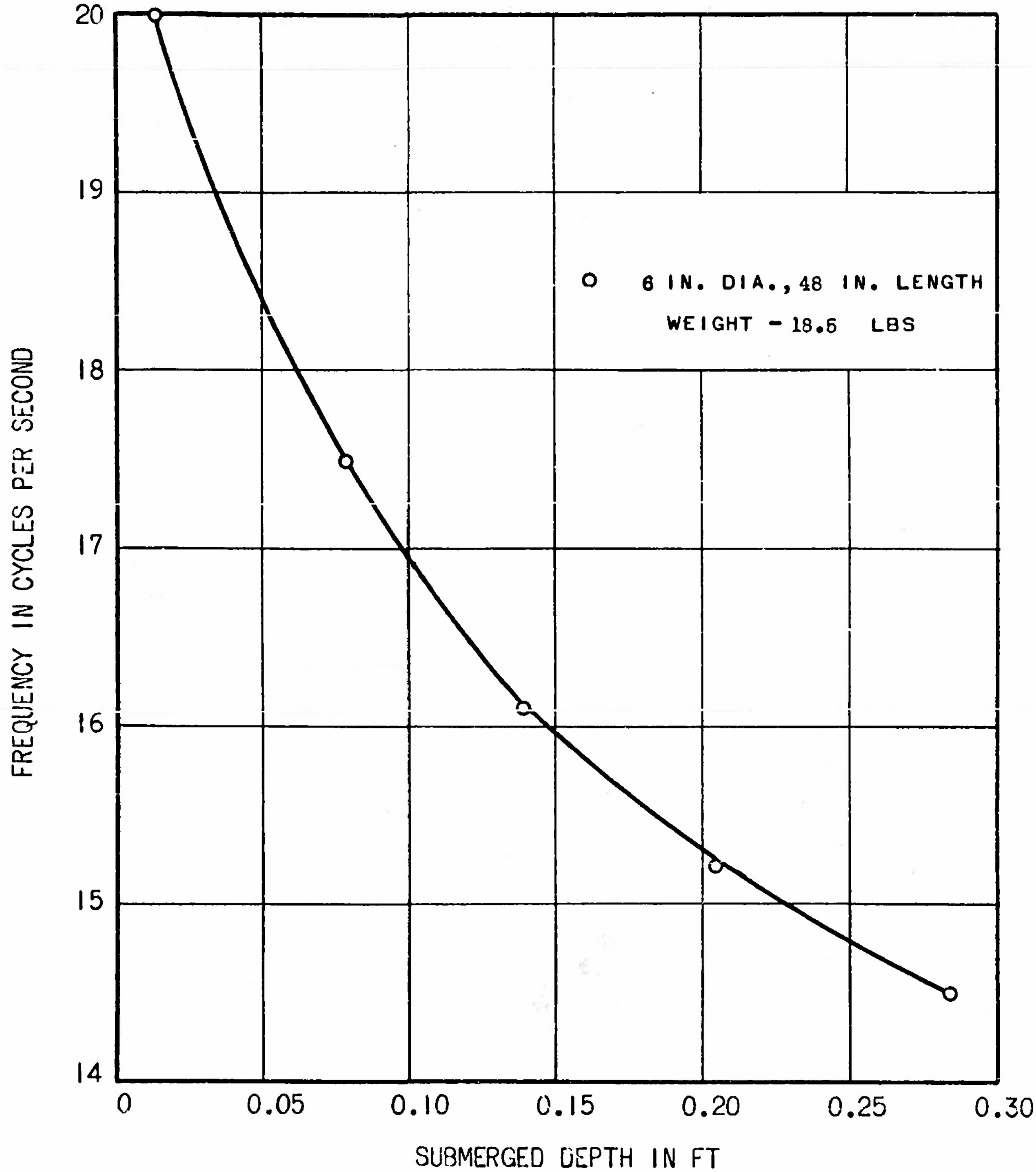

FIG. 18 FREQUENCY AND SUBMERGENCE FOR SEMICIRCULAR PONTON

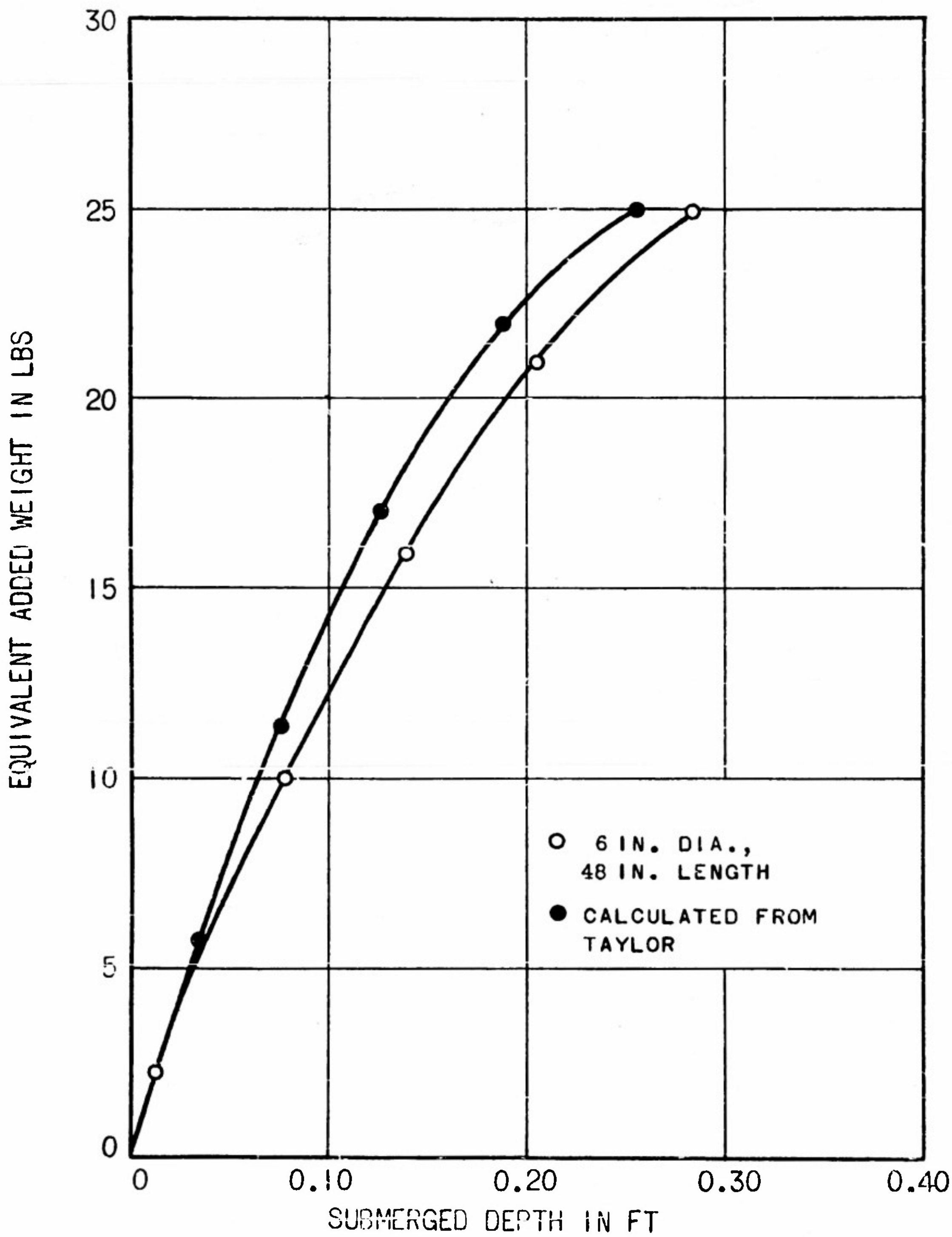

FIG. 19 EQUIVALENT ADDED WEIGHT AND SUBMERGENCE FOR SEMICIRCULAR PONTON

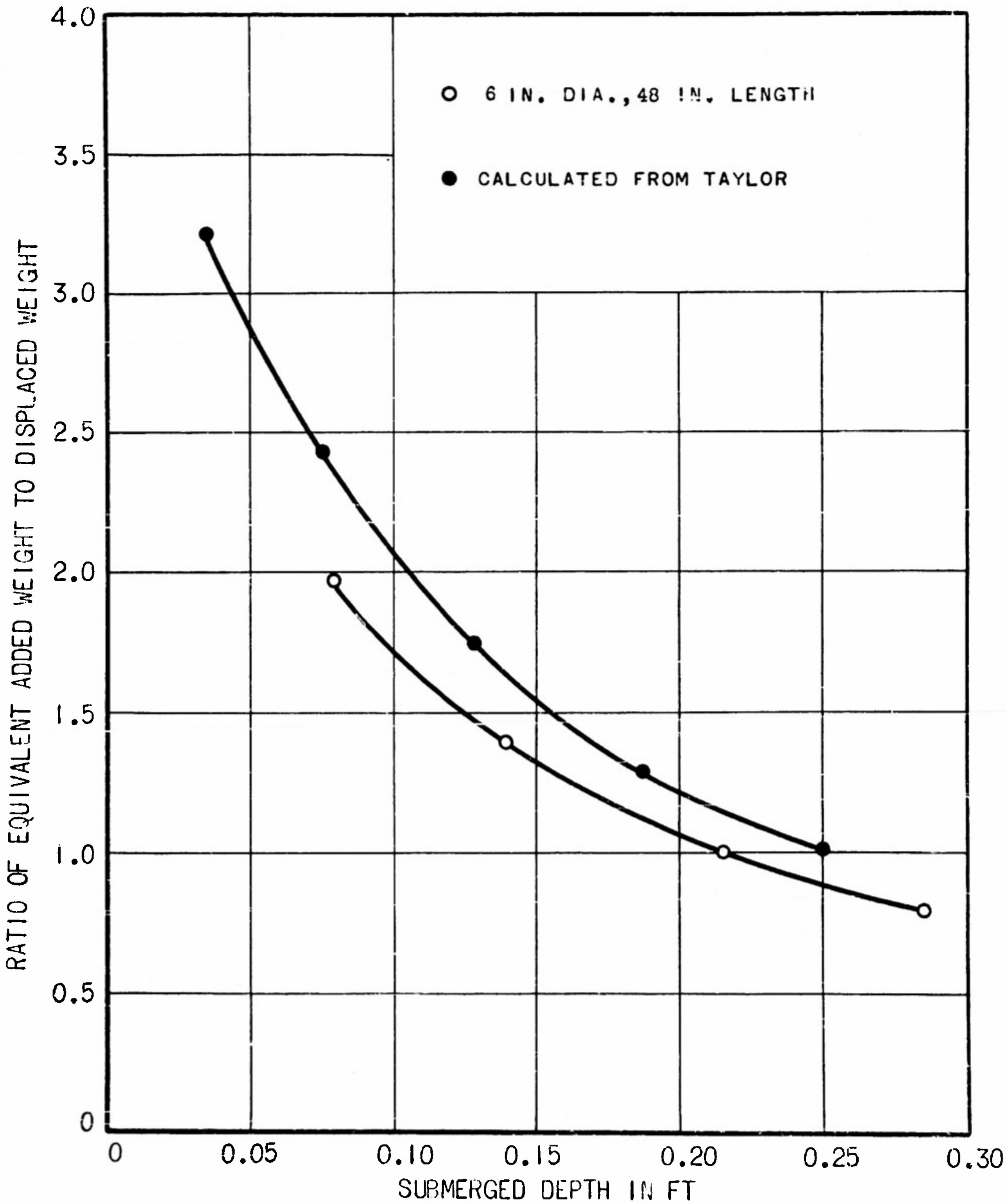

FIG. 20 RATIO OF EQUIVALENT ADDED WEIGHT TO DISPLACED WEIGHT AND SUBMERGENCE FOR SEMICIRCULAR PONTON

speeds and spacing should be specified to prevent resonance. The dynamic stresses that accompany a resonant vibration may be very much more than the 15 per cent allowed for impact.

The ponton would be primarily subject to two kinds of vibration. The first and simplest kind would be the vertical oscillation of the whole ponton about its equilibrium position. This is similar to the bobbing of a cork floating in water. The second kind of vibration would be the oscillation with two nodes such that the center of the ponton is down when the two ends are up and vice versa.

At 0.5 in. freeboard, the equivalent added weight of the one-twelfth scale model M-4 ponton is about 30 lbs. The corresponding quantities for the prototype would be (1/2) (12) = 6 in. freeboard and $(30)(12)^3$ = 51,800 lbs equivalent added weight. If the weight in a vacuum is 3500 lbs, the effective virtual weight is 51,800+3500 = 55,300 lbs. In the foot-pound-second system the virtual inertia is 55,300/32.2 = 1720 lb-sec^2/ft. The displacement-force constant for the prototype M-4 ponton at 6 in. freeboard is about 23,000 lbs/ft.

The bobbing frequency of a floating body may be computed from the relationship,

$$f = \frac{1}{2\pi}\left(\frac{k}{m+C}\right)^{1/2} \tag{17}$$

where $\underline{f}$ is the frequency in cycles/sec, $\underline{k}$ the unbalanced force-displacement ratio in lbs/ft, and (m+C) is the total virtual inertia of the body in lb-sec^2/ft. For the M-4 model ponton with 6-in freeboard

$$f = \frac{1}{2\pi}\left(\frac{23000}{1720}\right)^{1/2} = 0.581 \text{ cycles/sec.} \tag{18}$$

The bobbing period would be 1/0.581 = 1.72 sec.

The vibrational frequency of a free bar with two nodes may be computed from the expression

$$f = \frac{1}{2\pi} \quad (22.4) \quad \left[\frac{EI}{(m+C)L^3} \right]^{1/2} \qquad (19)$$

where E is the modulus of elasticity in lbs/sq ft, I the moment of inertia of the body about a horizontal transverse axis through the centroid in ft^4, and L the length of the bar in ft. This expression assumes the virtual inertia (m+C) to be uniformly distributed along the length L.

For the M-4 ponton assume E = 10,000,000 lbs/sq in. = 1,440,000,000 lbs/sq ft; I = 60,000 $(in.)^4$ = 0.289 $(ft)^4$. Let L = 56 ft, which is shorter than the overall length to correct for the rounded bow. Then

$$f = \frac{1}{2\pi} \quad (22.4) \quad \left[\frac{(1,440,000,000)\ (0.289)}{1720 \quad (56)^3} \right]^{1/2} \qquad (20)$$

$$= 4.18 \text{ cycles/sec} \qquad (21)$$

for the two node vibration of the M-4 ponton with 6 in. freeboard. The period of vibration would be 1/4.18 = 0.239 sec.

The attached mass of the superstructure would slightly reduce the frequency of the two node vibration. The unbalanced buoyant forces would slightly increase the frequency. Both of these factors are neglected.

Neglecting the added inertia due to the water would have erroneously increased both these frequencies by a factor of $(55,300/3500)^{1/2}$ = 3.98.

Both frequencies of 0.581 and 4.18 cycles/sec. are in the range of forcing vibration impulses from crossing vehicles and tread "knock." The added inertia should certainly be considered in an analysis of design and operation of floating bridges to prevent harmful vibration of the ponton.

Operational Behavior and Added Mass

Romualdi (11) has analyzed the effect of virtual mass on the operational behavior of a ponton bridge. He found that:

> "Virtual mass (of the pontons) has a significant effect on the response of floating bridges to transient loads. The effect is more pronounced than the effect of viscous damping. Moreover, the omission of virtual mass in the analysis of a floating bridge leads to displacements on the unsafe side."

Romualdi (11) has compared the measured and computed displacements of a ponton subjected to a known impulse. The measured and computed displacements agree very closely when added inertia and viscous drag are considered. If these quantities were neglected, the calculated results would be greatly in error.

Thus, the measured added inertia can be applied to typical transient loading problems to determine ponton behavior. The results of model tests can then be safely applied to problems in the design and operation of the prototype structures.

CONCLUSIONS

1. The measured added mass or inertia is reasonable and consistent with the results of previous experiments and with analyses of potential flow. The experimental method is simple and accurate.
2. The added inertia due to the fluid around a ponton may be many times the inertia of the ponton as measured in a vacuum. The equivalent added weight is usually greater and is often many times the displaced weight of water.
3. The added inertia due to the water is of prime importance in a study of the hydrodynamic properties of pontons. The added inertia has a very important effect on the operational characteristics of floating bridges.
4. The resonant frequencies for the two simplest kinds of vibration of a ponton are near the forcing frequencies of moving vehicle loads and tread "knock." Neglecting the added inertia due to the surrounding water makes the computed frequencies in error by a factor of 4.0.

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